

GCSE OCR

Computer Science
J277

2

Binary arithmetic and hexadecimal

Unit 2
Data representation



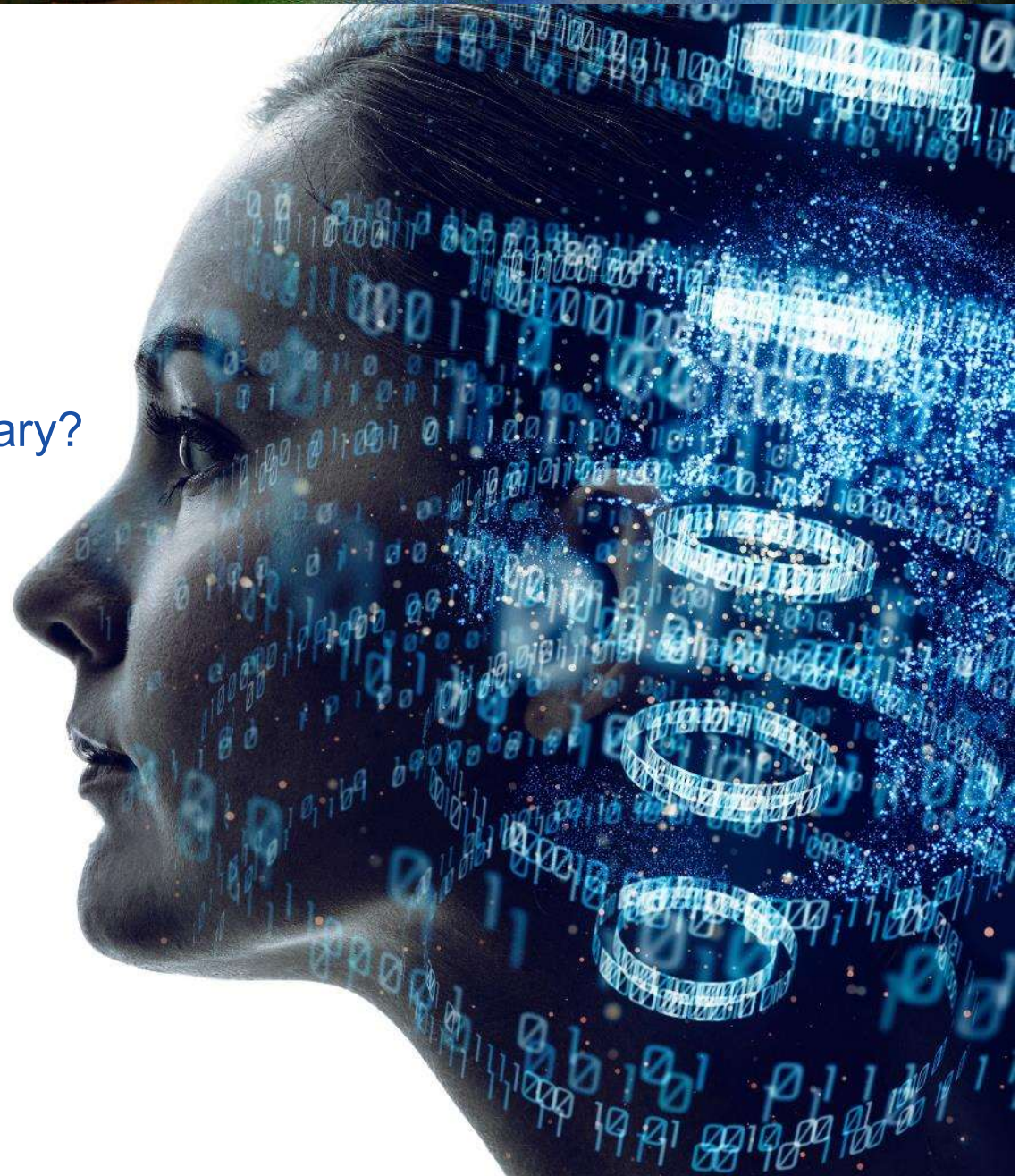
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Objectives

- Convert positive denary whole numbers (0-255) into 2-digit hexadecimal numbers and vice versa
- Convert between binary, denary and hexadecimal equivalents of the same number
- Add two 8-bit binary integers and explain overflow errors which may occur
- Understand the use of binary shifts

Starter

- Review:
 - What is 7 in binary?
 - What is 1001 in denary?
- What is:
 $1001 + 110$?



Starter

Answers

- What is 7 in binary?
 - 111
- What is 1001 in denary?
 - 9
- What is $1001 + 110$?

Binary	Denary
1001+	9+
110	6
1111	=15

Binary to denary conversion

- Practise converting binary to denary and back again
 - Convert each of the following to binary or denary

128	64	32	16	8	4	2	1
0	1	0	1	1	0	0	1
1	1	0	0	0	0	1	0

14

105

How can you quickly tell which is odd and even in any binary number?

Binary to denary conversion

Answers

- 0101 1001 = 89
- 1100 0010 = 194
- 14 = 1110
- 105 = 110 1001
- Even binary numbers will have a 0 at the end
- Odd binary numbers will have a 1 at the end

Hexadecimal

- Hexadecimal (or hex) is a number system which uses base 16
- As we only have 10 digits, it uses 0-9 and then the letters A to F

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

0 1 2 3 4 5 6 7 8 9 A B C D E F

- What is **F** in denary?
- What is **10** in hex?

Hex to denary conversion

- You will only need to translate one- or two-digit hexadecimal numbers

16s

Units

2

A

$$2 \times 16 + 10 = 42 \text{ in denary}$$

- Multiply the left-hand digit by 16, then add the units
- What is hex **27** in denary?

Denary to hex conversion

- Divide the denary number by 16 to get the number of 16s (the left-hand hex digit)
- The remainder gives you the units

Denary **18** becomes:

18 / 16 = 1 remained 2 so
the hex value for **18** is **12**

(Spoken, 'One Two', not 'Twelve')

- What is denary **27** in hex?
- What is denary **44** in hex?

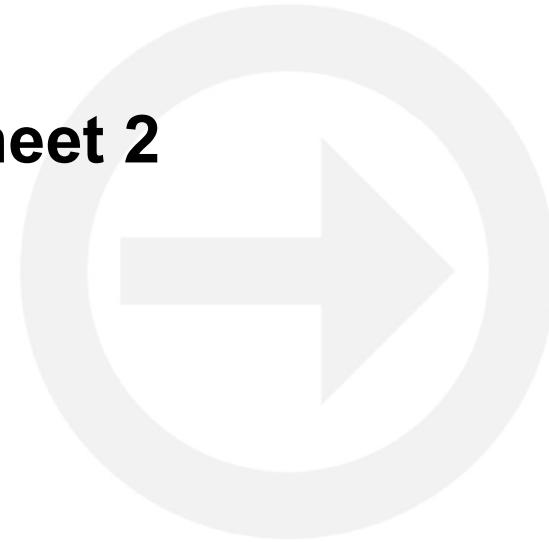
Hex to denary conversions

Answers

- What is hex 27 in denary?
 - $2 \times 16 + 7 = \underline{39}$
- What is denary 27 in hex?
 - 1×16 remainder 11 (11 is **B** in hex) = 1B
- What is denary 44 in hex?
 - 2×16 remainder 12 (12 is **C** in hex) = 2C

Worksheet 2

- Complete **Task 1a** on **Worksheet 2**



Binary to hex conversion

- Take a binary word of 8 bits

1 1 1 0 0 1 0 1

- Divide into two nibbles of 4 bits

1 1 1 0 0 1 0 1

- Convert each nibble into its hex value and rejoin

1 1 1 0 = 14 = E in Hex + **0 1 0 1 = 5** in Hex

So **1 1 1 0 0 1 0 1 = E5** in Hex

Hex to binary conversion

- What is **3B** in hex?

Split the two hex characters

3 = 0011 in binary and **B = 1011**

So **3B = 0011 1011** in binary

What is hex **21** in binary?

What is hex **A5** in binary?

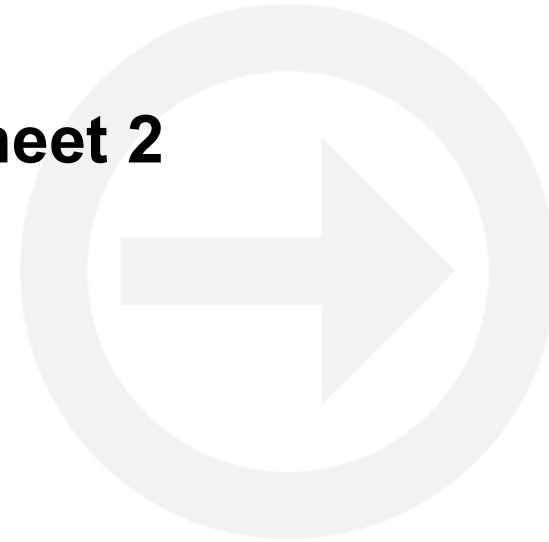
Hex to binary conversion

Answers

- What is hex 21 in binary?
 - $2 = 0010$, $1 = 0001$
 - 21 in hex = 0010 0001 in binary
- What is hex A5 in binary?
 - $A = 10 = 1010$, $5 = 0101$
 - A5 in hex = 1010 0101 in binary

Worksheet 2

- Complete **Task 1b** on **Worksheet 2**



Why use hex?

- There are advantages for programmers and Computer Scientists in using hex rather than binary
 - It is much simpler to remember a hex value than a binary value
 - It is quicker to write or type since a hex digit only takes up one digit rather than 4 bits
 - People are less likely to make an error with fewer digits
 - It is easy to convert between hex and binary

Binary addition

- Use binary to add two numbers together

$$1 + 1 = 10?$$

Simple denary addition

Work right to left :

1. Add the Units
2. If Over 9, *Carry* Tens
3. Add Tens

$$\begin{array}{r} 1 \\ 14 \\ + 17 \\ \hline = 31 \end{array}$$

The rules of binary addition

Work right to left and apply these simple rules:

1. $0 + 0 = 0$
2. $0 + 1 = 1$
3. $1 + 0 = 1$
4. $1 + 1 = 0$ *Carry 1*
5. $1 + 1 + 1 = 1$ *Carry 1*

	1	1				
		1	1	1	0	14
+		1	1	0	0	12
<hr/>						
=	1	1	0	1	0	26
	Carry Bit	Rule 5	Rule 4	Rule 2 or 3	Rule 1	

Adding binary values

$$\begin{array}{r} \text{Carry} \quad 1 \quad 1 \quad 1 \\ 1 \quad 0 \quad 1 \quad 0 \quad 1 \\ + \quad \quad 1 \quad 1 \quad 1 \\ \hline = 1 \quad 1 \quad 1 \quad 0 \quad 0 \end{array}$$

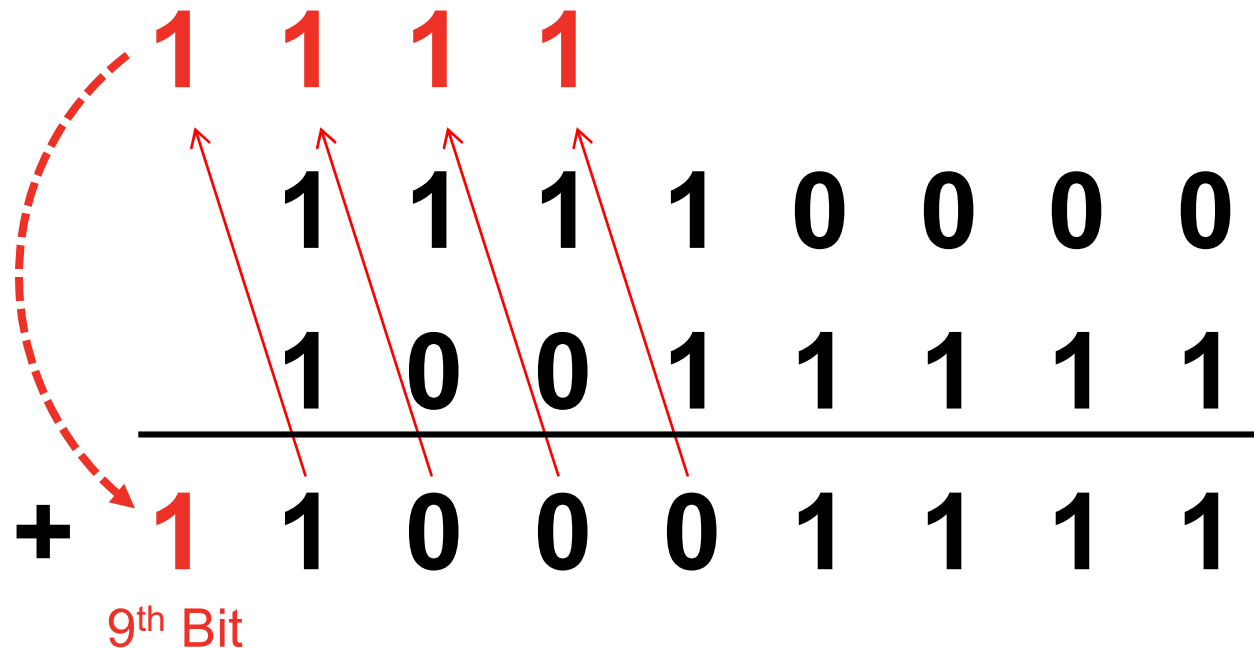
Adding numbers

- Computers work with a fixed number of bits at a time
 - This can cause problems
 - What problem will arise when adding the following bytes and storing the result in one byte?

$$\begin{array}{r} 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0 \\ +\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1 \end{array}$$

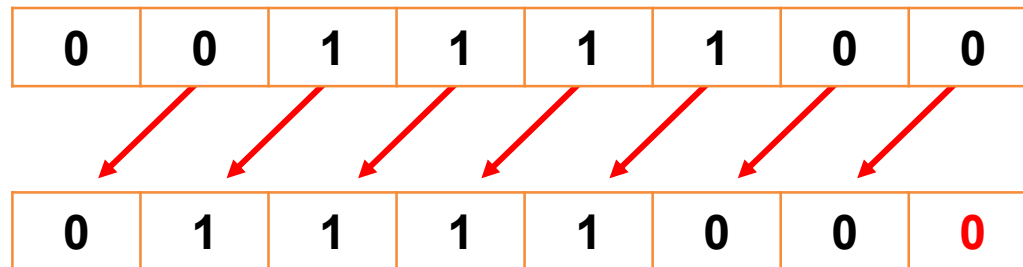
Overflow error

- When the result of an addition is too large for the number of bits the computer works with there will be an **overflow error**



Logical binary shift operations

- A binary shift left of one bit moves all the bits one place to the left
 - The vacant bit spaces are filled with zeros



- Looking at the table above, what effect does a shift left of one place have on the binary value?
 - What effect would a shift right of two places have?

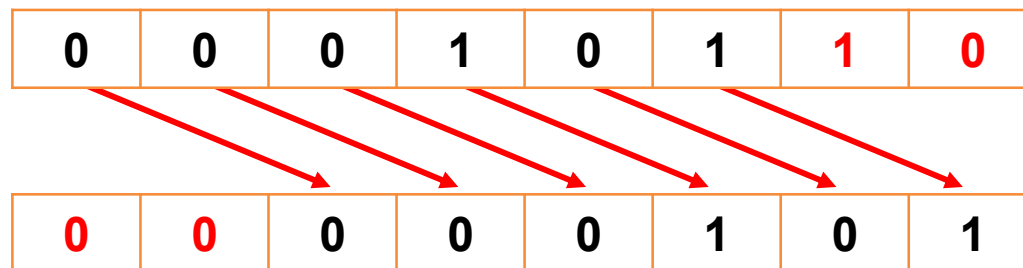
Shift operations

Answers

- A binary shift left of one bit doubles the number
 - E.g. $1000=8$
Shift left once, $10000=16$
- A binary shift right of two places results in halving the number and rounding down each time
 - Example 1: $1000=8$
Right shift once, $100 = 4$
Right shift again, $10 = 2$
 - Example 2: $1001 = 9$
Right shift once, $100 = 4$
Right shift again, $10 = 2$

Effects of shifts

- Logical shifts can very quickly multiply or divide a binary number by a factor of two
 - Left shifts multiply
 - Right shifts divide
- A loss of accuracy can occur if 1 bits are removed:
 - $22 / 4$ is not exactly 5



Worksheet 2

- Complete **Task 2**



Plenary

- Work out the answers to each of the following then compare your answers with a partner
 - Convert C in hexadecimal to denary
 - Convert 2B in hexadecimal to binary
 - Convert 1010 0011 in binary to hexadecimal
 - Calculate
$$\begin{array}{r} 1001\ 0101 \\ + \\ 0010\ 0110 \\ \hline \end{array}$$
 - Calculate the left shift of 0110 1011
 - Explain the effect of a left shift

Plenary

Answers

- C in hexadecimal = 12 in denary
- 2B in hexadecimal = $16 \times 2 + 1 \times B = 32 + 11 = \underline{43}$
- 1010 0011 in binary = A3 in hexadecimal
- $1001\ 0101 + 0010\ 0110 = \underline{1011\ 1011}$ in binary
- Left shift 0110 1011 (107) = 1101 0110 (214)
- The left shift doubles the number

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